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RESEARCH MEMORANDUM

BLENDING MODELLING IN A PROCESS MANUFACTURING SYSTEM

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Abstract

This paper describes the application of mixed integer programming to a blending problem in a process industry. The application is designed for the personal computer and is in use in France by a producer of chemical fertilizers. The paper first discusses the differences between the blending problems. It covers then the formulation of the blending problem of the plant, its solution and the successful implementation. Special consideration is given to the problem of handling the nonlinearity introduced by constraints of the process manufacturing system. Further consideration is given to the use of opportunity cost in the model.

Key Words: Blending, Optimization

1. Introduction

As Williams (1989) mentioned: "Mathematical Programming is one of the most widely used techniques in Operational Research. In many cases its application has been so successful that its use has passed out of operational research departments to become an accepted routine planning tool". The most important techniques belonging to mathematical programming are the Linear Programming (LP), the Mixed Integer Programming (MIP) and the Quadratic Programming (QP). Currently, there exist several optimization algorithms and computer packages for every technique. One of the important steps in Operations Research is to use and apply properly these techniques to real-life problems.

This paper is based on a real-life case study about the production of chemical fertilizers introduced by DSM. Here we discuss the application of mathematical programming to the blending problem in this company. Blending is a well known problem in process industries (see Taylor et al. (1981)). Examples can be found in the food industry (diet problem, Stigler (1945)), in the petroleum industry and in the chemical industry. In general, a blending problem requires the determination of the cheapest blend of available raw materials which meets certain (nutritional) requirements of the final products. What are known at the outset are the costs, nutritional composition and availability of the raw materials, and the requirements concerning the nutritional content of the final blends.

The goal of the study is to develop a user friendly microcomputer model to solve the blending problem. The major improvements involved are reduction in raw material costs, time required for planning, and more accurate tactical decisions to be made with respect to transfer prices of internally supplied raw materials.

It is evident that process industries differ fundamentally from assembly industries. Within a process industry there is a broad range of industrial activities present. This diversity is caused by the character of products, the type of process and the size of flexibility of the process installations. The flexibility of an installation describes whether an installation is only dedicated to one product or dedicated to more than one product. Most of the firms, belonging to process industries have a high degree of product differentiation.

In a process industry there is a fair uncertainty with regard to:

- *The output of the production process, expressed in quantity and quality*

  The quality of the output, which depends on the quality of the used raw materials and the process parameters (e.g. pressure and temperature), is often difficult to foretell.

- *The availability of raw materials*

  Mostly the production process is founded on a few different raw materials. If they are not available the production process stops. Therefore, the dependency on the suppliers is great. Besides, often the way of transport of raw materials leads into long leadtimes.

To reduce these uncertainties, process industries keep larger amounts of stock both of raw materials and final products compared to other industries. The stocks of raw materials are mostly large because of the relative low prices of the raw materials, the way of supply and the fast use of the raw materials. Process industries are more capital expensive than assembly industries. In addition, in process industries the capacity of the installations determine the rhythm of production and mostly cannot be enlarged by using overtime. Another difference between the two types of industries concerns the use of substitutes and the arising of by-products during the production process. The use of different formulas or recipe for the same product results from the substitution possibilities in process industries.

It is clear that the planning and control systems should be dedicated to the systems' characteristics in process industries. As such, application software are more system dependent. Table 1 provides an overview of some planning and control systems used in process industries. Some important characteristics of process industries will be discussed in section 3.

As mentioned before, this paper describes the development and implementation of a decision support system for the blending problem of DSM Engrais France. DSM Engrais France is a producer and seller of chemical fertilizers. This company belongs to the Agro business unit of the chemical company DSM. DSM Agro is one of the larger producers of chemical fertilizers in Europe. The headquarter of DSM Engrais France is founded in Beauvais, while the production takes place in Gouaix. DSM Engrais France sells not only its own chemical fertilizers on the French market, but sells also chemical fertilizers produced in the Netherlands. Via a distribution network, DSM Engrais France delivers the products to the customers. The total yearly sales equals FF 800 million of which approximately 40% fertilizers produced in Gouaix. The products and production process are described in section 3.

Section 2 gives a theoretical consideration of linear programming application to blending problems. Next in section 3 the blending problem of DSM Engrais France is described. In this section the problem is formulated as a MIP model. The personal computer system and the use of opportunity cost in the model, are also discussed here. Finally, in section 4 the paper is closed with some general conclusions.

Computer information systems (CIS) - order processing systems - inventory processing systems - capacity requirements planning (CRP) - material requirements planning (MRP) - production scheduling
Forecasting systems - moving average - exponentional smoothing - linear regression - subjective forecasting
Production planning systems - economic order quantity (EOQ) - simulation - linear and mixed integer programming - dynamic programming - capacity balancing
Inventory systems - ABC-analysis - MRP - statistical inventory control (SIC) - replenishment systems

Table 1. Planning and Control Systems Applied in the Process Industries

2. Blending Problems : A Theoretical Discussion

Blending problems can be divided into two categories: single-blend and multi-blend. Multi-blend problems attempt the minimization of the overall cost of producing several blends simultaneously subject to tie-in constraints to the total availability of common stocks of raw materials. Single-blend problems attempt to minimize separately the cost of each blend. In case of a constraint to the availability of one or more raw materials, that can be used in different blends (final products), a multi-blend problem should be considered.

To solve a blending problem, either a single-blend or a multi-blend, often a LP model can be developed and optimized on a microcomputer by use of an optimization package. The linear programming formulations (models) of blending problems can be found in most books on linear programming. A good description of LP modelling of blending problems are given in the papers by Wilson and Willis (1985) , by Munford (1989) and by De Kock and Sinclair (1987). In these LP models continuous variables are used to represent the quantities of the raw materials in the final blends. Blending problems has become a fairly routine application of linear programming.

Clearly, in practice blending problems are mostly more complicated in nature than the "standard" blending problem with only nutritional requirements. In addition to the nutritional requirements, there are requirements introduced by the manufacturing environment, incompatibilities between raw materials, available stocks of raw materials, etc. Some of these conditions can easily be formulated as extra linear constraints in a LP model. However, there are also conditions which cannot be formulated as linear constraints in one model (e.g. the incompatibility between two or more raw materials). By use of mixed integer programming, which includes both continuous and integer variables, such nonlinear constraints can frequently be dealt with. Therefore, the complexity of the models are case dependent.

A good reference book on model building in linear and mixed integer programming is written by Williams (1989). A good application of mixed integer programming to a blending problem is described by Glenn (1988). In the case study of this paper, which will be described in section 3, a MIP model is developed in order to formulate the nonlinear constraints of the blending problem.

The following "standard" blending problem will illustrate the difference between single-blend and multi-blend. Consider a producer of n different chemical fertilizers. A fertilizer is defined in terms of its nutrients of interest (e.g. nitrogen, phosphate, potash, etc.). A chemical fertilizer is manufactured by blending m different raw materials (also fertilizers) together to meet the requirements. Each raw material i ($i=1,..,m$), with costs c_i, contents a percentage a_{ik} of nutrient k ($k=1,..,K$). Each final blend j ($j=1,..,n$) should content at least b_{jk} per cent of nutrient k. Let q_i be the available stock of raw material i at the beginning of a new production cycle. Suppose that in this cycle the producer wants to manufacture p_j units of final blend j ($j=1,..,n$) by blending available raw materials. The problem is now to determine the optimal blends such that the costs are minimal, subject to the requirement constraints. This blending problem can be formulated as a multi-blend linear programming problem.

If the continuous variable x_{ij} represents the quantity of raw material i in blend j, then the linear programming model for the multi-blend problem is:

$$min \sum_{j=1}^{n} \sum_{i=1}^{m} c_i \, x_{ij}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = p_j \qquad j=1,..,n \qquad (1)$$

$$\sum_{i=1}^{m} a_{ik} \, x_{ij} \geq p_j \, b_{jk} \qquad j=1,..,n,\ k=1,..,K \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} \leq q_i \qquad i=1,..,m \qquad (3)$$

The first set of constraints (1) is the formulation of the requirement that the weight of each blend must equal the weight of the used raw materials. The second set of constraints (2) formulates the nutritional requirements. The third set of constraints (3) concerns the available common stocks of raw materials.

Suppose that for the above mentioned problem a single-blend model for each blend separately is formulated. If x_{ij} is the quantity of raw material i in blend j then the single-blend LP model for blend j is:

$$min \sum_{i=1}^{m} c_i \, x_{ij}$$

subject to

$$\sum_{i=1}^{m} x_{ij} = p_j \tag{4}$$

$$\sum_{i=1}^{m} a_{ik}\, x_{ij} \geq p_j\, b_{jk} \qquad k=1,..,K \tag{5}$$

$$x_{ij} \leq q_i \qquad i=1,..,m \tag{6}$$

The third set of constraints (6), concerning the availability of raw materials for blend j, do not include the quantities of raw materials used in other blends. Let x_{ij}^* ($i=1,..,m$) be the optimal quantities for blend j ($j=1,..,n$), then the total required quantity G_i of raw material i for a production cycle is given by:

$$G_i = \sum_{j=1}^{n} x_{ij}^* \tag{7}$$

Since the blends are optimized separately, it is possible that the total required quantity G_i exceeds the available stock q_i of raw material i. The solution would be infeasible. In such situation a multi-blend model should be formulated. However, when there are no constraints on the availability of common stocks of raw materials and other common constraints, the multi-blend model can be decomposed into a set of smaller single-blend models for each blend separately.

3. The Blending Case of DSM Engrais France

Problem description

DSM Engrais France is a producer of two types of chemical fertilizers: PK and NPK. A PK contents the primary nutrients phosphate (P) and potash (K), while a NPK contents also nitrogen (N). A PK or NPK is produced by blending (and granulating) raw materials, also fertilizers, which content the relevant nutrients. The production process is straightforward. The products are produced in great batches (often a run equals 1 or 2 weeks). Dry raw materials are imported into the production process by funnels, while wet raw materials are imported through pipes. After this, the raw materials are blended together according to a recipe. During the production moisture evaporates so that the weight of the imported raw materials is greater than the output of the production.

The objective of the study is to minimize the total cost of production through an optimal blending policy. The problem then is to determine the optimal recipes for each PK and NPK, i.e. the combination of raw materials which meets the nutritional requirements of the product. In table 2 the nutritional requirements and a recipe for a typical product (NPK 15.15.15) are given. Note, since moisture evaporates during the production, the weight of the raw materials is greater than 1 ton.

Table 2
Nutritional requirements and a recipe (formula) for the product NPK 15.15.15

nutritional requirements		*recipe for 1 ton NPK 15.15.15*	
nutrient N	min. 15.0 %	raw material a	240 kg
nutrient P	min. 15.0 %	raw material b	145 kg
nutrient K	min. 15.0 %	raw material c	205 kg
		raw material d	180 kg
		raw material e	250 kg
		raw material f	8 kg
		raw material g	3 kg
		Total	1031 kg

The recipe has no relevant affect on the production time. For each product a lot of different combinations are possible which would meet the nutritional requirements. In the former production method, before the implementation of the computer application, only 4 or 5 recipes for each product were used to determine, by "trial and error", the requirements. Every month, on the basis of new prices of raw materials, the cost of raw materials for each recipe and each product was calculated. When a product should be produced then the recipe with the lowest costs of raw materials was selected from the available recipe for that product. However, this recipe is not necessarily the optimal! As mentioned, a choice was made from only 4 or 5 recipes, while many different combinations are possible which would meet the nutritional requirements.

It is clear that in addition to the nutritional requirements, there are also constraints caused by the production process (technical requirements), such as the number of funnels. Since the recipes do not affect the costs of production, the relevant costs in the blending problem consist of only the used raw materials' costs. This leads to the following definition of the blending problem of DSM Engrais France:

Of which raw materials and in which quantities should each product be composed to meet the nutritional and technical requirements, such that the costs of raw materials are minimal?

In section 2 blending models were divided into two categories: single-blend and multi-blend. Depending on the situation, a multi-blend model can be decomposed into a set of smaller single-blend models. The situation of DSM Engrais France is characterized by:

- Continuous supply of raw materials which is guaranteed by logistics department.
- Demand seasonality which requires the products to be made to stock.

The make-to-stock production allows to change the production sequence of the products during the months prior to a high season. Make-to-stock nature of production also permits raw material shortage for short periods. Since there are no constraints on the availability of common stocks of raw materials and there are no other common constraints, the blending problem of DSM Engrais France can be formulated for each blend separately.

Let's describe the blending problem in detail. Each blend (fertilizer) is composed of a number of raw materials which content the required nutrients. The proportions of the nutrients in the raw materials determine the quality of the final blend. Each blend should meet the nutritional requirements and the technical constraints. The *nutritional requirements* of a certain blend specify the limits of the levels of the nutrients in the blend. The technical constraints can be divided into following six categories:

a) *The balance constraint.*
Since moisture evaporates during the production process, the weight of the final blend is smaller than the weight of the used raw materials for the blend.

b) *The inclusion rate constraints.*
Some raw materials are constrained in their inclusion rates. Both lower bounds and upper bounds for the raw materials in the blends are distinguished.

c) *The funnel constraint.*
The number of raw materials in the blend which are imported through funnels in the

process installation, is restricted by the number of funnels. Such a restriction can also exist for the other raw materials.

d) *The quantity constraints.*
The raw materials are weighted per ton of the final blend. For some raw materials a lower bound for their inclusion rates exists only if they are used in the blend. For example, if raw material i is used in the blend then minimal h_i kg per ton should be used. This restriction can rule out small quantities of raw materials in the blends.

e) *The incompatibility constraints.*
Some combinations of raw materials are not allowed.

f) *Other logical conditions.*
Logical conditions associated with relations between raw materials. For example, if raw material A and B are used in the formula, a certain quantity of raw material C has to be included.

The model

As mentioned before, the different blends (products) are treated separately. Since the blends are produced on the same process installation, we can formulate one single-blend model which can be used for all the blends by changing the model parameters. To model the logical conditions (e.g. technical constraint c, d, e and f) it is necessary to use binary variables. In the MIP model, continuous variables are used for the quantities of raw materials, while binary variables are associated with the modelling of the technical requirements. The constraints in the model ensure that the nutritional and technical requirements are satisfied. The objective function of the model involves minimizing the relevant cost to blend a batch of say 1 ton. Since the relevant costs consist only of the costs of raw materials, the objective function is taken as a linear function of the quantity of material blended. We assume there are m raw materials and that there are K nutrients of interest. The objective function is a linear combination of the used raw materials x_i ($i=1,..,m$), weighted by their costs c_i:

$$\text{minimize} \qquad \sum_{i=1}^{m} c_i \, x_i \tag{8}$$

If amounts (in kg) x_i of the raw materials are combined, then the amounts (in kg) of the nutrients in the blend are:

$$\sum_{i=1}^{m} a_{ik}\, x_i \qquad\qquad k=1,..,K \tag{9}$$

where a_{ik} is the weight percentage of nutrient k in raw material i.

There will be nutritional requirements specifying that the weight percentage of the k-th nutrient in the blend must lie between limits b_k (%) and d_k (%), say, so we have K pairs of nutrient constraints:

$$b_k\ 1000 \le \sum_{i=1}^{m} a_{ik}\, x_i \le d_k\ 1000 \qquad\qquad k=1,..,K \tag{10}$$

Note that we may have one-sized constraints which means that b_k may be zero or d_k may be infinite (or both). For equality constraints we put $b_k=d_k$.

The balance condition (technical constraint (a)), which takes account of the evaporation of moisture, can be formulated by putting the weight of the used raw materials exclusive moisture on the same level with the dry matter content of 1 ton of the final blend. In the model, the percentage w of moisture in the final blend is supposed to be independent of the used raw materials. Let w_i be the percentage of moisture in raw material i, then the balance condition can be formulated as follows:

$$\sum_{i=1}^{m} (1-w_i\ /100)\ x_i = (1-w\ /100)\ 1000 \tag{11}$$

Note that (100%-w_i) is the dry matter percentage of raw material i.

The condition concerning the levels of the raw materials in the final blend (technical constraint b), is formulated by the following m pairs of constraints:

$$l_i \le x_i \le u_i \qquad\qquad i=1,..,m \tag{12}$$

The lower bound l_i is introduced in order to force certain raw materials to enter into the blend.

To formulate the condition concerning the number of funnels (technical constraint c), we must introduce binary variables to indicate whether or not a raw material is used in the blend:

δ_i	:=	1	if raw material i is used in the blend
	:=	0	otherwise

The relations between the continuous and binary variables for the raw materials, are formulated by the following m pairs of constraints:

$$m_i\ \delta_i\ \leq\ x_i\ \leq\ M_i\ \delta_i \qquad\qquad i=1,..,m \qquad (13)$$

M_i is a upperbound for x_i and m_i is a very small positive value. Having formulated these relations between x_i and δ_i, it is easy to formulate technical constraint c:

$$\sum_{i\epsilon I_F} \delta_i\ \leq\ N_F \qquad (14)$$

In the above equation, I_F represents the set of raw materials which are imported through the funnels, while N_F represents the number of funnels.

The quantity constraint (technical constraint d) can be formulated as follows:

$$x_i\ \geq\ h_i\ \delta_i \qquad\qquad i=1,..,m \qquad (15)$$

where h_i represents the minimal quantity (kg/ton) of raw material i in case of use. Note that the sets of constraints (12) and (15) can be combined with the set of constraints (6) by choosing the right values for the parameters m_i and M_i.

Clearly, in the real and more extended model of DSM Engrais France, the technical constraints (e) and (f) are also included. These conditions are formulated by use of new defined binary variables.

The model can be solved using any commercial MIP software. In this case the model is solved by the OMP optimization package (Beyers & Partners(1991)), which uses the branch and bound algorithm of mixed integer programming. Because the model should be easy to use, the personal computer system was set up as interactive and menu-driven. We can distinguish two main functions of the system. Firstly, the optimization function to determine the least-cost blends. Secondly, the database function to maintain the necessary data files such as the nutrient files, raw materials files, price files, etc. The user can easily change the data by using the built-in data editor. The system stores also the most recent compositions of the optimized blends. Another function of the system is to analyze the sensitivity of the blends for price changes. In addition, the system is an excellent tool for modifying the coefficients in the constraints, such as the number of funnels. By increasing the number of funnels by one, the manager can decide on basis of a new blending optimization if he should invest in a new funnel in the future, or to prepare a premix.

The results

The system is currently being used by DSM Engrais France[1], and the user is quite satisfied with the turnaround. The most important benefits of the system are given in table 3.

Table 3
Major benefits of the MIP model and the system

1	Cost reduction of approximately 2 or 3 per mill of the total raw material costs
2	Time reduction in calculation of formulas for new products of approximately 90 per cent
3	A decision support for purchasing and investments
4	Flexibility and user friendliness of the menudriven system

The cost reduction of 2 or 3 per mill of the total raw material costs, equals more than FF 500,000. Another major saving of the system is the time reduction in planning of the blend compositions. In the past, the production manager needed 1 hour to calculate a recipe, while the system determines the cheapest blend within 5 minutes (depending on the number of raw materials, the computer capacity, etc.). In addition, the system is a helpful tool to estimate the required raw materials in the future (purchasing planning) and is also a good support for investment decisions; for example the decision to invest in a extra funnel to increase the maximum number of dry raw materials allowed in the blends, or to invest in greater recipe flexibility. The system is also used to analyze the benefits of considering an alternative raw material for production. In case of a positive result of such an analysis, the new raw material is tested in a small production batch. Finally, the menu-driven system allows the user to perform sensitivity analysis. The user can easily obtain important information by changing the prices, the number of raw materials, etc. The system is designed so that a production manager with no

[1] In addition, a similar system for chemical fertilizers is developed by the authors and implemented by another DSM Agro production unit. This system includes also the optimization of production lot sizes using a MIP model.

knowledge of linear or mixed integer programming could use it with confidence. In fact, the user does not need to be aware that he is using a mixed integer programming package. Summarized, the benefits of the system exceed largely the costs of the system.

Opportunity cost discussion

Neglecting opportunity cost results in inefficient use of resources and loss of business opportunities. Good use of opportunity cost incorporates market forces within a management process of an integrated manufacturing system. It provides an incentive to each manufacturing plant to be as efficient as its successful independent competitors and supports sound decisions in productivity and resource allocation. When an optimization model of the type discussed here is used, it is very important to update and use proper data to make the right decisions. Cost figures are key elements in the data files. Therefore, the use of opportunity cost is elaborated here to highlight its importance in global decisions to be made.

DSM Engrais France is a part of the DSM Agro Business Unit. On behalf of the production, DSM Engrais France purchases its raw materials from both external suppliers and other production units belonging to the DSM Agro Business Unit. When a raw material is supplied by an external supplier, the market price is used in the model. However, when a raw material is supplied by a unit belonging to DSM Agro, special attention should be paid to the determination of the raw material price, which is to be used in the model. For such a type of raw materials transfer prices are charged by the supplying unit. The transfer price of an internally supplied raw material is the price which DSM Engrais France has to pay to the supplying unit of DSM Agro. If DSM Engrais France would use this transfer price in the model, it will optimize its own profit (cost). However, it will not necessarily optimize the business unit profits as a whole. Optimizing DSM Agro profits is the main managerial task. Therefore, the question is; "How to optimize group results and to avoid suboptimization?". In other words; "Which prices need to be used in the model for internally supplied raw materials?".

The main raw material for DSM Agro is gaz, which is transformed into ammonia and nitric acid. Based on this nitric acid, DSM Agro produces several straight fertilizers (only containing nitrogen N) and mixed fertilizers like NP's and NPK's in one or more production steps. Nitric acid is the basic raw material for all these types of fertilizers. Two situations are distinguished here:

1) **The nitric acid plants produce at full capacity.**

In this case, the raw materials that are internally supplied to DSM Engrais France have to compete with other fertilizers. Every ton of nitric acid which is transformed into a raw material for DSM Engrais France, implies one ton of nitric acid less to be transformed in one of the other applications/fertilizers. So the name of the game is to optimize the gross margin on nitric acid. In other words, the price to be used in the MIP-model is not the transfer price to be paid by DSM Engrais France, but the opportunity cost of that raw material. This opportunity cost can be derived from variable production costs of nitric acid, the additional variable production costs in the steps to transform the nitric acid in fertilizers or raw materials for DSM Engrais France, the transport costs to Gouaix, and the gross margin, forgone by DSM Agro if not producing a fertilizer but a raw material for DSM Engrais France. This opportunity cost is not necessarily equal to an eventual market price of the raw material which is internally supplied. If the opportunity cost is higher than the external market price, DSM Engrais France has to purchase its raw material at the market, regardless of the transfer price. Thus, the MIP-model price has to be fixed at the external market price. If not, group results will not be optimized. However, if the opportunity cost is lower than the market price, DSM Engrais France is better off to purchase the raw material internally, at whatever the transfer price may be. Then, the MIP-model price has to be fixed at the opportunity cost level. If not, again, group results will not be optimized.

2) **The nitric acid plants do not produce at full capacity.**
In this case, opportunity cost for the raw material to be supplied to DSM Engrais France is equal to the variable production and transport costs since only the variable costs are economized if the material is not produced. In this case, again, the opportunity cost has to be compared with the market price. Conclusions are similar to those before. However, normally the opportunity cost is lower than the market price (if not, the market price would even not cover the variable costs). Therefore, DSM Engrais France will have to purchase its raw material internally, at whatever transfer price may be, in order to optimize group results. Note that a difference between market price and opportunity cost is mainly due to market imperfections. To these market imperfections, fiscal constraints are added in transfer price setting. Thus, many possible cases exist in real business situations.

The above discussion is further made clear by the following example for the product NPK 17.17.17. This fertilizer is the main product of the Gouaix plant with a yearly production of about 75,000 ton. In table 4, three different optimized formulas for this product are shown. Each formula being an optimal formula, depending on the price of raw material (a) used in the model:
- transfer price of FF 600 per ton (standard case)
- opportunity cost of FF 635 per ton, exceeding the transfer price (case 1)
- opportunity cost of FF 450 per ton, inferior to the transfer price (case 2).

Table 4

Production receipts for NPK 17.17.17 under different prices for raw material (a)

raw material	standard case*	case 1*	case 2*
a	274	120	329
b	69	-	-
c	370	274	351
d	-	314	-
e	-	-	53
f	283	283	283
g	16	6	-
h	8	8	8
i	3	3	3
total	1023	1008	1027
transfer price raw material (a)	600		
opportunity cost raw material (a)		635	450
unit cost NPK 17.17.17 at transfer price 600**	861.20	861.75	864.20
unit cost NPK 17.17.17 at opportunity cost 635**	870.80	865.95	
unit cost NPK 17.17.17 at opportunity cost 450**	820.10		814.85

* : raw materials in receipts are given in kg per ton final product
** : price/cost of 1 ton of raw material (a)

As to be expected, in case 1 the quantity of raw material (a) to be used in the optimal formula is lower than the quantity of raw material (a) in the standard case; whereas in case 2 the quantity of raw material (a) to be used is larger than to the quantity in the standard case.

In order to increase insight in these alternative cases, the unit cost of NPK 17.17.17 per ton of the three formulas under the three possible prices of raw material (a) are calculated. The optimal unit cost at *transfer price* level is FF 861.20 per ton. For the standard formula, the FF 870.80 per ton is found at *opportunity cost* level of FF 635 per ton of raw material (a).

This is calculated as follows: 861.20+(635-600)*0.274=870.80, i.e. optimal unit cost at transfer price level plus the difference between opportunity cost and transfer price times the amount of raw material (a). For case 1, the optimal solution is FF 865.95 per ton. This formula at *transfer price* level results in FF 861.75 unit cost, which is determined as follows: 865.95+(600-635)*0.120=861.75. Evidently, at *transfer price* level, the standard formula is found cheaper by the MIP model (FF 861.20 per ton). However, at the opportunity cost of FF 635 per ton, the standard formula results in a higher unit cost for the group DSM Agro.

In case 1, if management in Gouaix decides to produce the case 1 formula in stead of the standard formula, the economy for the group DSM Agro will be 870.80-865.95, i.e. FF 4.85 per unit (based on a yearly production of 75,000 tons, this makes FF 360,000). However, raw material (a) is still purchased at transfer price, this decision implies a loss for the plant in Gouaix, because they do not produce anymore their "suboptimal" standard formula. This loss for Gouaix adds up to 861.75-861.20=0.55 FF per ton, or FF 40,000 per year. So, the supplying company of DSM Agro is making an additional profit of FF 5.40 per ton, or FF 400,000 per year. The FF 5.40 per ton should be interpreted as follows; the supplying company will supply less of raw material (a), that is 274-120=154 kg per ton of NPK 17.17.17. The reduction of supply of raw material (a), makes it possible to transform the nitric acid into another fertilizer, which generates a higher revenue. The opportunity cost of raw material (a) exceeds the transfer price with 635-600=35 FF per ton. So the additional revenue for the supplying company can be found from: (0.274-0.120)*(635-600)=0.154*35=5.40 FF per ton.

Similarly for case 2 we find:

- economy for DSM Agro: 820.10-814.85= 5.25
- disadvantage for Gouaix: 864.20-861.20= 3.00
- additional revenue for the supplying company: 8.25

Note that: (0.329-0.274)*(600-450)=0.055*150=8.25.

In words; the supplying company delivers 329 kg instead of 274 kg of raw material (a) per ton of NPK 17.17.17. Opportunity cost of raw material (a) is FF 450 per ton and is transferred at

FF 600 per ton. Thus, the supplier is making an additional profit of FF 8.25 per ton of NPK 17.17.17. The total yearly economy for DSM Agro amounts approximately FF 400,000.

From these examples we may conclude that the use of opportunity cost instead of transfer price may preserve the company from suboptimization in both cases; i.e. whatever opportunity cost is, larger than the transfer price or lower than it.

4. General conclusions

In this paper an application of the mixed integer programming to a blending problem in the process industry was described. The blending problem in the case DSM Engrais France is an extension of the classical blending problem. The use of binary variables were necessary to model the technical constraints caused by the process manufacturing system. The problem consisted of the minimization of the overall costs of the manufacturing of chemical fertilizers. Because of the situation of DSM Engrais France, i.e. no constraints on the availability of common stocks of raw materials and other common constraints, the formulation of a complex multi-blend model was not necessary. The blending problem could be decomposed into a set of smaller single-blend problems. Since all blends are manufactured on the same process installation, we needed to formulate only one single-blend model which can be used for all the blends by changing the model parameters.

The model is solved by implementing the OMP optimization package for linear and mixed integer programming. The model has been incorporated as a part of a larger user-friendly system. The system was set up as interactive and menu-driven so that users (managers) with no knowledge of mixed integer programming could use the model with confidence. One of the facilities which proved useful was the ability to calculate quickly optimal blend compositions and costs to meet special customer desires. The system, which is installed on a personal computer, has been received extremely well by DSM Engrais France. The most important benefits of the system are: cost reduction, time reduction in planning, helpful planning tool for investment and purchasing decisions. The benefits of the system exceed largely the costs of the system.

The use of correct data is a major task in building a mathematical programming model. Once this has been done the organization should be geared to providing and updating the data regularly. We showed that it is important to use opportunity cost in the model to ensure optimal decisions for the organization as whole.

References

1. De Kock, H.C., and M. Sinclair, (1987), *Multi-mix feedstock problems on microcomputers*, Operational Research Quarterly, **38**, p. 585.

2. Eijs, A.G.M. van (1991), *Twee toepassingen van de gemengd geheeltallige programmering in de procesindustrie*, research paper.

3. Glenn, J.J., (1988), *A mixed integer programming model for fertilizer policy evaluation*, European Journal of Operational Research, **35**, p. 165.

4. Munford, A.G., (1989), *A microcomputer system of formulating animal diets which may involve liquid raw materials*, European Journal of Operational Research, **41**, p. 270.

5. OMP optimization package 4.0, an optimization package for linear and mixed integer programming, Brasschaat (Belgium): Beyers & Partners.

6. Stigler, G.J., (1945), *The cost of subsistence*, Journal of Farm Economics, **27**, p. 303.

7. Taylor, S.G., Seward S., Bolander S., and Heard R., (1981), *Process Industry Production and Inventory Planning Framework: A Summary*, Production and Inventory Management Journal, Vol. 22, No. 1, p. 15-32.

8. Williams, H.P., (1989), *Model building in Mathematical Programming*, Wiley.

9. Willis, R. and E. Wilson, (1985), *Microcomputers and linear programming - feedstock revisited*, European Journal of Operational Research, **19**, p. 297.

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